A SPHERICAL RADIOMETER

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A theoretical analysis is performed of a radiation detector constructed in the form of a spherical cavity within which a filament heater is located, with an externally mounted thermopile.

Radiant flux measurements may employ either receivers or radiators with known characteristics. At the present time, however, it is felt that radiators cannot perform measurements with uncertainties of less than 1% [1, 2].

Metrological studies employ receivers in the form of various-shaped cavities with the measured flux reproduced by electrical heating. The nonequivalence of the action of the flux and the electrical power is the basic error source in such radiometers. An attempt to eliminate this error source was undertaken in [3] by integrating the thermal effect over the radiometer surface, independent of the source of that effect. The radiometer is a cone wound of wire, the resistance of which varies upon heating. The form of that radiometer is obviously not optimal, since absorption and radiation conditions are different along the directrix.

In the radiometer produced by the author, with the form of a Wood horn, the difference in indications upon irradiation by a helium–neon laser at the peak and base of the cavity reached 5%.

Another possible approach is integration of the flux density radiated into space over the radiometer surface:

$$P=\int_{s} q ds. \tag{1}$$

We replace the integral by a summation and express the flux by Newton's formula

$$P=\sum_{i=1}^{N}\alpha_i (T_i - T_0)\,\Delta s. \tag{2}$$

The coefficient of heat liberation in a vacuum depends on the temperatures T_i and T_0 and the properties of the surface. However, if the surface is homogeneous and the relative change in temperatures T_i is small (Fig. 1), we may then assume that $\alpha_i = \alpha =$ const and define the mean temperature value from the system of equations

$$\alpha=\varepsilon\sigma(T^2+T_0^2)(T+T_0),\ T=T_0+P/4\pi R^2\alpha.$$

We divide the entire surface into N equal size areas $s_i = 4\pi R^2 N^{-1}$, then sum $T_i - T_0$, which can be done by a thermopile, the hot junctions of which are located in the i areas, with cold junctions at temperature T_0. The thermopile emf will then be

$$U=kN/4\pi R^2\alpha.$$

A similar expression relates the emf to the electric heater power: the measured flux can be determined from the expression $P = UME^{-1}$. To eliminate the effects of thermopile nonlinearity, E must be close to M. This last relationship is all the more accurate, the closer the sum of Eq. (2) is to the integral of Eq. (1), i.e., the greater the number N. However, technological difficulties limit the possible number of thermocouples. The situation may be improved by making use of the symmetry of the radiometer temperature field. The thermocouples may be located in the plane of symmetry with a variable spacing such that each thermopair belongs to a toroidal zone lying in a plane perpendicular to the axis of symmetry with area [4] $s_i = 2\pi R^2(1 + \cos\beta)/N$. The outer boundary of the i-th zone (inner boundary of the first zone β) is given by the expression

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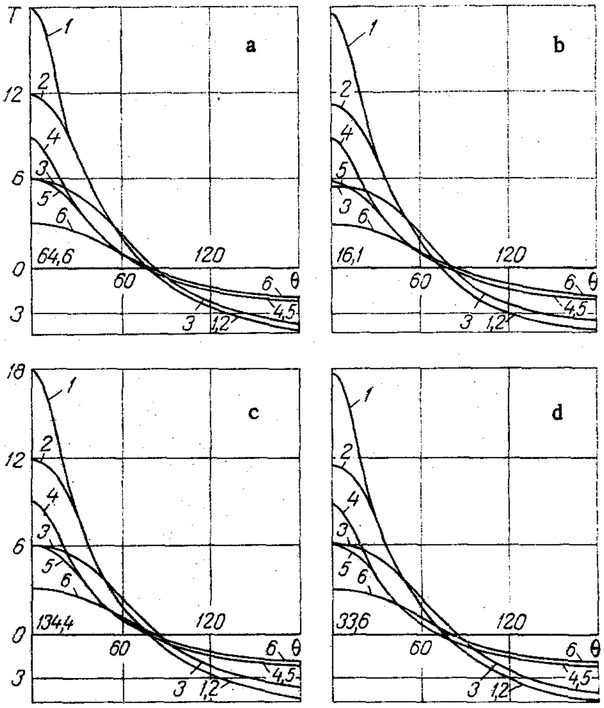

Fig. 1. Surface temperature of spherical radiometer vs latitude at P = 1 W, λ = 350 W/m·deg K. Zero value of the ordinate is average temperature shift $\bar{t} = P/4\pi R^2\alpha$. Curves 1, 2, 3 correspond to wall thicknesses $\delta = 0.5 \cdot 10^{-4}$ m; 4, 5, 6, $\delta = 1 \cdot 10^{-4}$ m; 1 and 4, θ = 15°; 2 and 5, θ = 30°; 3 and 6, θ = 60°; a) $R = 2 \cdot 10^{-2}$ m; α = 3.08 W/m²·deg K; b) $R = 4 \cdot 10^{-2}$ m; α = 3.08 W/m²·deg K; c) $R = 2 \cdot 10^{-2}$ m; α = 1.48 W/m²·deg K; d) $R = 4 \cdot 10^{-2}$ m; α = 1.48 W/m²·deg K. $\bar{t}$, °C.

$$\cos\theta_i = \cos\beta\left(1 - \frac{i}{N}\right) - \frac{i}{N}. \tag{3}$$

The lower limit of the number N must be established from the admissable nonequivalence of measured flux and electrical power. For its definition we must consider the temperature field developed within the radiometer.

According to [5], at the moment of measurement the stationary temperature field on the outer surface of the radiometer has the form

$$T = T_0 + \frac{P}{4\pi R^2\alpha} + \frac{P\varepsilon}{4\pi r^2(1-x_0)} \sum_{m=1}^{\infty} \frac{(2m+1)^2 W_m(x_0) L_m(x)}{\frac{R^{m+1}}{r^{m+2}}(m+1)(m\lambda+\alpha R) - \frac{r^{m-1}}{R^m} m[(m+1)\lambda - \alpha R]}. \tag{4}$$

Integration of Eq. (4) over the radiometer surface ($ds = 2\pi R^2 \sin\theta d\theta$; $\sin\theta d\theta = -dx$), in view of the orthogonality of the Legendre polynomials, produces a family

$$\int_s \alpha(T - T_0)\,ds - P = 0. \tag{5}$$

The third term of Eq. (4), which may be termed $T(\theta)$, must pass through a null at some $\theta°$, otherwise the law of conservation of energy, of which Eq. (5) is an expression, will be violated.

On curves of $T(\theta)$ constructed by calculation with Eq. (4) on a Minsk-32 computer, it is evident that $\theta°$ varies little with change in parameters of either radiometer or flux (change in P produces no effect, while variation in x_0 shifts $\theta°$ insignificantly), and is located in the range between θ = 75 and 80°. It is also evident that, because $\alpha R \lambda^{-1} \approx 10^{-4}$, change in α has practically no effect on the curve. The tangential temperature gradient is approximately three orders of magnitude greater than the radial. It is somewhat unexpected that upon increase in sphere radius the curves show practically no decrease while the tangential temperature gradient $\mathrm{grad}_\theta T$ decreases in inverse proportion to the change in radius: $\mathrm{grad}_\theta T = \Delta T/R\Delta\theta = \mathrm{const}/R$, so that the thermal flux passing through the section with coordinate θ is independent of radius:

$$Q_\theta = 2\pi R \sin\theta\delta\left(-\lambda\frac{\partial T}{R\partial\theta}\right) = -2\pi\lambda\sin\theta\frac{\partial T}{\partial\theta}.$$

To obtain the relationship between the number N and the error produced by replacement of integral (1) by sum (2), we find the absolute error proportional to the double sum:

$$\eta = \sum_{i=1}^{N}\sum_{m=1}^{\infty}\left(a_m R^m + \frac{b_m}{R^{m+1}}\right)L_m(x)\cos\theta_i, \tag{6}$$

where $\cos\theta_i$ should be taken from Eq. (3).

The relative error, as follows from Eqs. (2), (4), is

$$\mu = \left(\frac{R}{r}\right)^2 \frac{\alpha\varepsilon}{1-x_0}\frac{\eta}{N}. \tag{7}$$

Computer calculations with Eqs. (6), (7) permit the following conclusions.

The relative error decreases approximately proportionally to wall thickness and the square root of the number of thermocouples, and inversely proportionally to the heat liberation coefficient and the square of the sphere radius, i.e., $\mu = C\alpha R^2\delta^{-1}N^{-\frac{1}{2}}$. S is of the order of magnitude of 10^{-3} m·deg K/W at λ = 350 W/m·deg K, and is independent of θ_0.

Values of μ for α = 1.48 W/m^2·deg K, $\delta = 0.5 \cdot 10^{-4}$ m, N = 90 are as follows: for θ_0 equal to 15, 30, 60°, μ is 0.82, 0.58, 0.32%.

This last expression reveals methods for reducing the replacement error. First of all, the angle of divergence of the radiation beam entering the radiometer must be increased. As is evident, increase in the beam half-angle from 15 to 60° reduces the error by a factor of 2.6. However, there are technical difficulties in beam modification, connected with absorption and aberration of wide angle beams, which lead to an increase in flux measurement error.

The heat liberation coefficient should be reduced. This can be done, first, by cooling the radiometer, e.g., by placing it in a cooling screen (as described in [4]), so that thermal noise also decreases; second, the emissivity of the radiometer surface should be decreased by constructing it of low emissivity material (nickel, for example) and polishing.

Increase in radiometer wall thickness also reduces the error.

The latter two approaches lead to an increase in radiometer thermal inertia, and aside from increasing the time needed to perform measurements, this can introduce errors connected with the limited stability of the radiator over time.

Most effective is reduction in the sphere radius, since error is proportional to the square of the radius; the mechanical strength of the radiometer also increases in this case. But decrease in radius reduces the upper limit of measurable flux. Moreover, difficulties in radiometer construction and application due to corresponding reduction in input orifice size will increase.

Least effective is increase in the number of thermocouples, but this approach at least has no negative consequences like the preceding ones.

We will now consider factors not appearing explicitly in the expression for relative substitution error. These are the effects of the thermal conductivity coefficient and the coefficient of reflection of the inner surface (cavity) of the radiometer.

The reflection coefficient of the internal surface determines equalization of the radiation field inside the cavity by multiple rereflection. It appears in Eq. (4), where on the basis of Kirchhoff's law it is replaced by the emissivity of the cavity material. Upon reduction in the reflection coefficient (a silver coating has ε = 0.01-0.02) the dependence of surface temperture on angle θ decreases together with the substitution error; however, the absorbing capability of the cavity then falls, which leads to an increase in measurement error (for a nonradiating heater) in accordance with the expression

$$\mu = (1-\cos\beta)(1-\varepsilon)/2\varepsilon.$$

Thus, with the exception of the thermal conductivity coefficient, the value of which is in essence limiting, and the number of thermocouples, change in all other radiometer parameters directed toward reducing the substitution error will degrade other performance characteristics, and these various parameters must be chosen in accordance with the radiometer's intended use.

NOTATION

P, radiant flux; q, thermal flux; s, surface area; α, heat liberation coefficient; i, number of radiometer zone; T_i, temperature of i-th zone; T_o, temperature of surrounding medium; r and R, radii of inner and outer surfaces of radiometer; N, number of zones; U and E, thermo-emf of thermocouple during flux measurement and flux substitution; k, thermoelectric coefficient of thermocouple; M, electrical power of substitute heater; β, entrance orifice half-angle; ε, emissivity of cavity material; θ, angular coordinate in spherical coordinate system; $x = \cos\theta$, θ_o, and x_o correspond to boundary of area irradiated by flux; $L_m(x)$, Legendre polynomial of first sort and m-th order; $W_m(x_o) = \int_{-1}^{1} L_m(x)dx$; λ, thermal conductivity coefficient of radiometer material; θ^o, latitude at which curve $T(\theta)$ passes through zero; δ, cavity wall thickness; Q, thermal flux; μ and η, relative and absolute error in flux measurement; C, proportionality coefficient; σ, Stefan–Boltzmann constant.

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CONDUCTIVITY OF NONUNIFORM SYSTEMS

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Percolation theory and methods of generalized conduction theory are used to construct a model of a heterogeneous system and to determine the effective conductivity.

Statement of the Problem. We consider a very simple two-component heterogeneous system with random distribution of components, consisting of two kinds of identical isomeric particles, occupying the entire system volume without voids (Fig. 1a).

We need to find the effective conductivity Λ of the system (the thermal and electrical conductivity, the dielectric constant, the magnetic permeability, the diffusion, etc.) as a function of the conductivities Λ_i and the volume concentrations m_i of the i-th component when the latter do not interact, i.e., the quantity Λ_i does not depend on the concentration m_i.

The effective conductivity Λ of this system is determined from the equation

$$\langle \mathbf{j} \rangle = -\Lambda \langle \nabla\varphi \rangle, \tag{1}$$

where $\langle \mathbf{j} \rangle$ is the average flux over the volume V (heat, electricity, material, etc.) and $\langle \nabla\varphi \rangle$ is the average volume gradient of the potential due to the flux $\langle \mathbf{j} \rangle$:

$$\langle \mathbf{j} \rangle = \frac{1}{V}\int_V \mathbf{j}(\mathbf{r})\,dV, \quad \langle \nabla\varphi \rangle = \frac{1}{V}\int_V \nabla\varphi(\mathbf{r})\,dV. \tag{2}$$

Here for the local fluxes $\mathbf{j}(\mathbf{r})$ and the potential gradients $\nabla\varphi(\mathbf{r})$ we have the equations

$$\begin{gathered} \mathbf{j}(\mathbf{r}) = -\Lambda(\mathbf{r})\nabla\varphi(\mathbf{r}), \\ \operatorname{div}\mathbf{j}(\mathbf{r}) = 0, \\ \operatorname{curl}\nabla\varphi(\mathbf{r}) = 0. \end{gathered} \tag{3}$$

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